## Homework 6 (Graded)

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## Due Friday, 7/19, by the end of Sheridan's office hours at 4pm in CMU B-023

ASV exercises 4.54 and 5.40 (and state a general theorem relating $M(a)$ to $M(b)$ for positive $a<b--y o u$ do not need to prove it. Hint: search for "Jensen's inequality").

S6.1: consider the example from class, in which we assume there are 10,000 people of child-bearing age in Puyallup, that they become pregnant independently each with some probability $p$. Suppose that 9 months later we will check the birth records for these 10,000 people. Find an upper bound for the probability that the observed percentage of people who gave birth differs from $p$ by more than 0.02 ( 2 percentage points). Then, find the birth rate that would yield the highest probability for the observed number of births under the binomial distribution with $n=10,000$. This birth rate $q$ should be a function of the observed number of births.

S6.2: construct the following examples (must be nontrivially different than those discussed in class):

- A sequence of random variables that converges in distribution to another random variable, but not in probability.
- A sequence of random variables that converges in probability to another random variable, and prove it.

S6.3: Let $X \sim \operatorname{Unif}(0,2 \pi), Y=\sin (X)$. Find the pdf of $Y$.
S6.4: Suppose you are using a programming language that permits you to define arbitrary functions, but as for randomness can only generate Unif( 0,1 ) random variables. You must generate samples from the following random process: Sheridan flips a fair coin. If heads, he picks an angle uniformly at random from 0 to 45 degrees, shoots a harpoon at a 100 m high wall from 100 m away (no gravity), and measures how high on the wall the harpoon landed. If tails, he picks an $X$ and a $Y$ each uniformly at random from -1 to 1 , plots the point, and determines which quadrant (1-4) of the Cartesian plane it lands in. How do you generate such a sample? You may use "pseudocode," words, and mathematics to describe the process, but your final explanation need only be 1) fairly short and 2) correct.

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Points 15
Submitting on paper
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| Due | For | Available from | Until |
| :--- | :--- | :--- | :--- |
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